***PROBABILITY THEORY***

***Summary:***

***1.*** *Probability theory deals with events of chances.*

***2.*** *An event is a single result of an experiment*

***3.*** *A set of all possible outcomes of an experiment is called a sample space or possibility space*

***4.*** *Probability of an event* ***=*** **

***5.*** *Probabilities can be expressed as fractions, decimals or percentages*

***6. (i)*** *The probability of an event lies between* ***0*** *and* ***1***

***(ii)*** *The probability of an impossible event is* ***0***

***(iii)*** *The probability of a sure event is* ***1***

***7. (i)*** *The probability of event* ***A*** *occurring is written as* ***P(A).***

***(ii)*** *The probability of event* ***A*** *not occurring is written as or***

***(iii)*** *For any event* ***A,******P(A) +*** ** ***= 1*** *or * ***= 1 − P(A)***

***EXAMPLES:***

***1.*** *An integer between* ***1*** *and* ***10*** *inclusive is chosen at random****.*** *Find the probability that the chosen integer is****:***

***(i)*** *more than* ***4***

***(ii)*** *an even number*

***(iii)*** *a multiple of* ***3***

***(iv)*** *a triangular number*

***2.*** *A box contains* ***3*** *red****,******5*** *green and* ***12*** *blue pens****.*** *Find the probability that the pen picked from the box is****:***

***(i)*** *red*

***(ii)*** *green*

***(iii)*** *blue*

***(iv)*** *not green*

***(v)*** *not blue*

***(vi)*** *either red or blue*

***3.*** *A box contains red****,*** *green and blue pens****.*** *The probability of picking a blue pen from the box is  and that of a green pen is  Find the probability of picking a red pen*

***4.*** *A box contains* ***8*** *red pens and the rest are blue****.*** *The probability of picking a blue pen from the box is  Find the number of blue pens in the box*

***5.*** *A fair coin is thrown once****.***

***(a)*** *Write down its sample space*

***(b)*** *Find the probability of obtaining****:***

***(i)*** *a head*

***(ii)*** *a tail*

***6.*** *Two fair coins are tossed together****.***

***(a)*** *Write down its sample space*

***(b)*** *Find the probability of obtaining****:***

***(i)*** *two heads*

***(ii)*** *a head and a tail*

***7.*** *Three fair coins are tossed simultaneously****.***

***(a)*** *Write down its sample space*

***(b)*** *Find the probability of obtaining****:***

***(i)*** *exactly two heads*

***(ii)*** *at least two heads*

***8.*** *A fair dice is thrown once****.***

***(a)*** *Write down its sample space*

***(b)*** *Find the probability of obtaining****:***

***(i)*** *an even number*

***(ii)*** *a square number*

***(iii)*** *either a* ***3*** *or a* ***4***

***9.*** *Two fair dice are thrown together****.***

***(a)*** *Draw a table for the possible outcomes*

***(b)*** *Find the probability that****:***

***(i)*** *both show even numbers*

***(ii)*** *both show similar faces*

***(iii)*** *one shows an even number and the other odd*

***(iv)*** *the sum of the scores is* ***9***

***10.*** *Two fair dice are thrown simultaneously****.*** *Find the probability that the sum of the scores is*

***(i)*** *five*

***(ii)*** *less than five*

***(iii)*** *greater than six*

***11.*** *A fair coin and a fair die are thrown together****.*** *Find the probability of obtaining**a tail and an even number*

***12.*** *Two fair coins and a fair die are thrown together****.*** *Find the probability of obtaining****:***

***(i)*** *two heads and a number greater than* ***3***

***(ii)*** *at least one tail and a number less than* ***4***

***13.*** *A two digit number is formed using the digits* ***2, 3*** *and* ***4*** *without repeating any digit in the number formed****.***

***(a)*** *Write down the possibility space for the numbers formed*

***(b)*** *Find the probability that the number formed is****:***

***(i)*** *even*

***(ii)*** *less than* ***30***

***14.*** *A two digit number is formed using the digits* ***0, 3*** *and* ***5*** *without repeating any digit in the number formed****.***

***(i)*** *Write down the possibility space for the numbers formed*

***(ii)*** *Find the probability that the number formed is**less than* ***53***

***Soln:***

***Hint: 03*** *and* ***05*** *are non−two digit numbers since* ***03*** *is the same as* ***3*** *and* ***05*** *is* ***5***

***15.*** *A three digit number is formed using the digits* ***3, 4*** *and* ***5*** *without repeating any digit in the number formed****.***

***(i)*** *Write down the possibility space for the numbers formed*

***(ii)*** *Find the probability that the number formed is**even*

***16.*** *The weather forecast says the probability that it will be wet today is  and that of tomorrow is *

***(i)*** *Copy and complete the probability tree diagram below****:***

***today***

***tomorrow***

***wet***

***dry***

***wet***

***wet***

***dry***

***dry***





***(ii)*** *Find the probability that it will be wet on just one of the two days*

***17.***  *A coin is biased so that the probability of tossing a head is  If it is tossed three times****,***

***(a)*** *Draw a tree diagram showing the possible outcomes*

***(b)*** *Find the probability of obtaining****:***

***(i)*** *exactly two heads*

***(ii)*** *at least two tails*

***(iii)*** *two tails on the first two tosses*

***18.*** *A box contains* ***5*** *red and* ***3*** *blue pens****.*** *Two pens are drawn in succession at*

*random from the box with replacement****.***

***(a)*** *Draw a tree diagram showing the possible outcomes*

***(b)*** *Find the probability of picking****:***

***(i)*** *pens of the same colour*

***(ii)*** *exactly one red pen*

***(iii)*** *at least one red pen*

***(iv)*** *at most one blue pen*

***19.*** *A box contains* ***8*** *red and* ***3*** *blue pens. Two pens are drawn in succession at*

*random from the box without replacement.*

***(a)*** *Draw a tree diagram showing the possible outcomes*

***(b)*** *Find the probability of picking****:***

***(i)*** *pens of different colours*

***(ii)*** *exactly one red pen*

***(iii)*** *at least one red pen*

***(iv)*** *at most one blue pen*

***20.*** *A bag contains* ***5*** *red****, 7*** *blue and* ***8*** *green beads. Three beads are drawn in succession at random from the bag without replacement.*

***(a)*** *Draw a tree diagram showing the possible outcomes*

***(b)*** *Find the probability of picking****:***

***(i)*** *three green beads*

***(ii)*** *one bead of each colour*

***(iii)*** *beads of the same colour*

***21.*** *A bag contains* ***30*** *white,* ***20*** *blue and* ***20*** *red balls****.*** *Three balls are drawn in*

*succession at random without replacement. Find the probability that the first*

*ball and third ball is white.*

***EER:***

***1.*** *An integer between* ***10*** *and* ***30*** *inclusive is chosen at random****.*** *Find the probability that the chosen integer is****:***

***(i)*** *prime*

***(ii)*** *divisible by* ***2, 3*** *or* ***5***

***(iii)*** *a triangular number*

***(iv)*** *a factor of* ***240***

***2.*** *A letter is chosen from the word* ***“MISCELLANEOUS”.*** *Find the probability that it will be****:***

***(i)*** *an* ***S***

***(ii)*** *a vowel*

***(iii)*** *a consonant*

***3.*** *In a game****,*** *a player tosses a fair coin once****.*** *When a* ***1*** *or* ***6*** *appears the player wins and when a* ***3*** *or* ***4*** *appears the player losses****.*** *Find the probability that the player neither wins nor losses****.***

***4.*** *A two digit number is formed using the digits* ***1, 2, 4, 5*** *and* ***7*** *without repeating any digit in the number formed****.***

***(a)*** *Write down the possibility space for the numbers formed*

***(b)*** *Find the probability that the number formed is****:***

***(i)*** *divisible by* ***5******(ii)*** *divisible by* ***2*** *or* ***5 (iii)*** *greater than* ***50***

***5.*** *Two fair dice are thrown together****.*** *Find the probability that the product of the scores is*

***(i)*** *twelve*

***(ii)*** *four*

***6.*** *A fair coin is tossed three times****.***

***(a)*** *Draw a tree diagram showing the possible outcomes*

***(b)*** *Find the probability of obtaining****:***

***(i)*** *three heads*

***(ii)*** *exactly two heads*

***(iii)*** *at least two tails*

***7.*** *Two fair dice are thrown together****.*** *The score is the positive difference of the two numbers on which the dice land****.***

***(a)*** *Construct a table showing the possible outcomes*

***(b)*** *Find the probability that the positive difference of the scores is*

***(i)*** *two* ***(ii)*** *five*

***8.*** *A box contains* ***15*** *red pens and the rest are blue****.*** *The probability of picking a red pen from the box is  Find the****:***

***(i)*** *probability of picking a blue pen*

***(ii)*** *number of pens in the box*

***9.*** *A bag contains* ***3*** *white and* ***5*** *black balls****.*** *If two balls are picked in succession*

*at random from it, find the probability that the second ball picked is white if*

*picking is done****:***

***(i)*** *with replacement.*

***(ii)*** *without replacement.*

***10.*** *Three balls are drawn at random one after the other without replacement from*

*a bag containing* ***4*** *white****,******8*** *blue* ***, 5*** *red and* ***3*** *pink balls. Find the probability*

*that the first ball is blue****,*** *the second red or blue and the third is white.*

***11.*** *A box contains two types of balls* ***red*** *and* ***black****. When a ball is picked*

*from the box, the probability that it is red is  Two balls are selected at*

*random from the box without replacement. Find the probability that****:***

*the second ball is black*

***12.*** *A box contains* ***3*** *red,* ***4*** *green, and* ***5*** *blue beads. Two beads are selected at*

*random from the box without replacement. Find the probability of picking****:***

***(i)*** *beads of the same colour*

***(ii)*** *at least one red bead*

***(iii)*** *beads of different colours*

***13.*** *A fair die and a fair octahedral die are thrown together and the outcome on each die is recorded****.***

***(a)*** *Draw a table showing the possible outcomes*

***(b)*** *Find the probability that****:***

***(i)*** *both show odd numbers*

***(ii)*** *both show similar faces*

***(iii)*** *one shows an even number and the other odd*

***(iv)*** *the sum of the scores is greater than or equal to* ***5***

***(v)*** *the product of the scores is* ***4*** *or* ***12***

***14.*** *A bag contains* ***x*** *red balls and* ***(x − 8)*** *blue ones****.*** *The probability of picking a red ball from the box is* ***0⋅75.*** *Find the****:***

***(i)*** *probability of picking a blue ball*

***(ii)*** *number of balls in the bag*

***15.*** *A box contains* ***7*** *red and* ***5*** *blue pens. Two pens are drawn in succession at*

*random from the box without replacement.*

***(i)*** *Copy and complete the probability tree diagram below****:***

***1st draw***

***blue***

***blue***



***red***

***red***



***blue***

***red***

***2nd draw orrow***

***(ii)*** *Find the probability of picking at least one red pen*

***16.*** *A fair die and a fair tetrahedral die are thrown together and the outcome on each die is recorded****.***

***(a)*** *Draw a table showing the possible outcomes*

***(b)*** *Find the probability that****:***

***(i)*** *both show odd numbers*

***(ii)*** *the sum of the scores is greater than or equal to* ***5***

***(iii)*** *the product of the scores is* ***4*** *or* ***12***

***17.*** *Two fair dice are thrown together****.***

***(a)*** *Draw a table for the possible outcomes*

***(b)*** *Find the probability that****:***

***(i)*** *both show even numbers*

***(ii)*** *both show similar faces*

***(iii)*** *one shows an even number and the other odd*

***(iv)*** *the sum of the scores is* ***9***

***18.*** *Two fair tetrahedral dice each with faces numbered* ***1*** *to* ***4*** *are thrown together****.*** *The score is the sum of the two numbers on which the dice land****.***

***(a)*** *Construct a table showing the possible outcomes*

***(b)*** *Find the probability that the sum of the scores is*

***(i)*** *seven*

***(ii)*** *less than six*

***19.*** *Two fair dice are thrown together****.*** *Find the probability that the positive difference of the scores is*

***(i)*** *two*

***(ii)*** *five*

***20.*** *A box contains* ***5*** *red and* ***3*** *blue balls. Two balls are drawn in succession at*

*random from it without replacement. Find the probability that****:***

***(i)*** *they are of the same colour*

***(ii)*** *the second ball drawn is red.*

***21.*** *A box contains* ***5*** *red and* ***3*** *blue balls. Three balls are selected in succession at*

*random from it without replacement. Find the probability that****:***

***(i)*** *they are of the same colour*

***(ii)*** *the first and last are of the same colour*

***(iii)*** *at most one blue ball is drawn.*

***22.*** *Two fair tetrahedral dice are thrown up at once****.*** *Find the probability that the sum of the scores on the dice is less than* ***6.***